M10/11H Challenging 5 Assignment Quadratic Problems:

1. Find all possible values of a/b if $a^2 + 4b^2 = 4ab$

2. Let $f(x) = a^2x^2 + 5ax + 3$ and f(2) = 2. Find all possible values of the constant "a"

3. Find the value of "x" if "x" is positive and x-1 is the reciprocal of $x+\frac{1}{2}$

4. Let "f" be a function for which $f(x/3) = x^2 + x + 1$. Find the sum of all the values of "z" for which f(3z) = 7 [amc12]

5. Let "a" and "b" be the roots of the equation $x^2 - mx + 2 = 0$. Suppose that $a + \frac{1}{b}$ and $b + \frac{1}{a}$ are the roots of the equation $x^2 - px + q = 0$. What is the value of "q"?

6. Find all real solutions to $(x^2 - 5x + 5)^{x^2 - 9x + 20} = 1$

7. Find all solutions to the system of equations:

$$x^2 + yz = 39$$

$$x - yz = -33$$

$$y + z = 12$$

8. Find the roots of $x^2 + \left(a - \frac{1}{a}\right)x - 1 = 0$ in terms of "a"

9. Find the solutions to $(x^4 - 11x^3 + 24x^2) - (4x^2 - 44x + 96) = 0$

10. If $\frac{x^2 - bx}{ax - c} = \frac{m - 1}{m + 1}$ has solutions for "x" such that each solution is the negative of the other, then find "m" in terms of "a" and "b". [AHSME]

11. Find constants "a" and "b" such that b-a is as small a possible, and the entire graph of the equation

$$y = \frac{1 - x^2}{1 + x^2}$$
 lies within $a < y \le b$

12. Prove that if $\frac{a+b}{a} = \frac{b}{a+b}$ then "a" and "b" can't both be real numbers

13. Let "m" and "n" be the roots of $ax^2 + bx + c = 0$. Prove that if $m^2 + n^2 = 1$, then $2ac = b^2 - a^2$